# Simulation of Beam Instabilities in SPring-8

T. Nakamura
JASRI / SPring-8

nakamura@spring8.or.jp
http://acc-web.spring8.or.jp/~nakamura/

## Beam Instabilities

Multi-bunch

- Long range wake >> $T_b$ (bunch spacing)
  - Resonator(cavity)
  - Resistive-wall
  - Ion
  - Electron Cloud,...

Single-bunch

- Shot range wake << $T_b$
  - Geometrical (MAFIA)
  - Resistive wall (Analytical)
  - Electron Cloud
  - CSR

# Multi-bunch instabilities by Resonator
## Transverse

$$Z = \frac{\omega_r}{\omega} \frac{R}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Transverse Kick Voltage produced by a charge

$$\lambda = i\alpha + \omega_r'$$

$$V_j\left(t - t_j\right) = q_j x_j \frac{R}{Q} \omega_r \frac{\omega_r}{\omega_r'} e^{-\alpha\left(t - t_j\right)} \sin \omega_r'\left(t - t_j\right) = q_j x_j \,\mathrm{Re}\left[W e^{i\lambda\left(t - t_j\right)}\right]$$

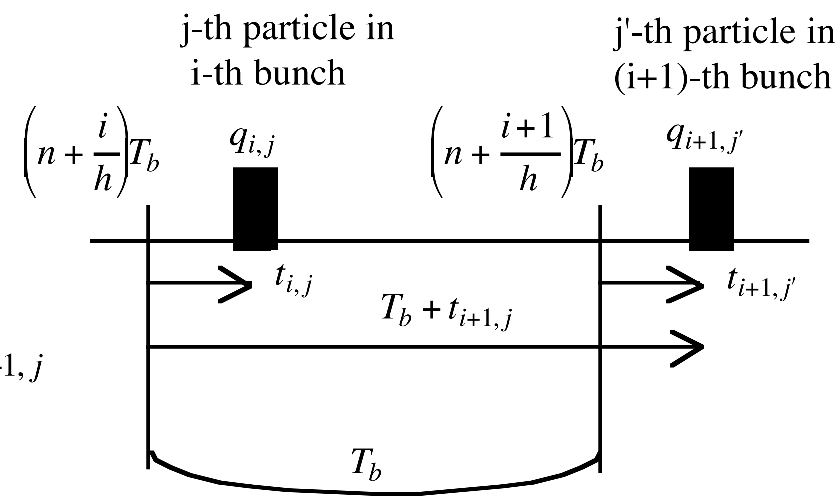
After (i+1)-th bunch,

$$W = -i \frac{R}{Q} \omega_r \frac{\omega_r}{\omega_r'}$$

$$V_{i+1}(t) = \mathrm{Re}\left[\tilde{V}_{i+1} e^{i\lambda t}\right] = \mathrm{Re}\left[\tilde{V}_i e^{i\lambda\left(T_b + t\right)} + \sum_{j=1}^{N_{i+1}} q_{i+1,j} x_{i+1,j} W e^{i\lambda\left(t - t_j\right)}\right]$$

$$= \mathrm{Re}\left[\left(\tilde{V}_i e^{i\lambda T_b} + \sum_{j=1}^{N_{i+1}} q_{i+1,j} x_{i+1,j} W e^{-i\lambda t_j}\right) e^{i\lambda t}\right]$$

$$\tilde{V}_{i+1} = \tilde{V}_i e^{i\lambda T_b} + \sum_{j=1}^{N_{i+1}} q_{i+1,j} x_{i+1,j} W e^{-i\lambda t_{i+1,j}}$$



# Multi-bunch instabilities by Resonator
# **Longitudinal**

$$Z = \frac{R}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Acceleration Voltage produced by a charge

$$\lambda = i\alpha + \omega_r'$$

$$V_j\left(t - t_j\right) = -q_j \frac{R}{Q} \omega_r e^{-\alpha\left(t - t_j\right)} \left(\cos \omega_r'\left(t - t_j\right) - \frac{\alpha}{\omega_r'} \sin \omega_r'\left(t - t_j\right)\right) = q_j \operatorname{Re}\left[W e^{i\lambda\left(t - t_j\right)}\right]$$

$$W = -\frac{R}{Q}\omega_r\left(1 + i\frac{\alpha}{\omega_r'}\right)$$

After (i+1)–th bunch,

$$V_{i+1}(t) = \operatorname{Re}\left[\tilde{V}_{i+1} e^{i\lambda t}\right] = \operatorname{Re}\left[\tilde{V}_i e^{i\lambda\left(T_b + t\right)} + \sum_{j=1}^{N_{i+1}} q_{i+1,j} W e^{i\lambda\left(t - t_{i+1,j}\right)}\right]$$

$$\tilde{V}_{i+1} = \tilde{V}_i e^{i\lambda T_b} + \sum_{j=1}^{N_{i+1}} q_{i+1,j} W e^{i\lambda\left(-t_{i+1,j}\right)}$$

# Beam Loading

$\tilde{V}_b$ { Acceleration Mode = Longitudinal Impedance
Excitation ⇧
Beam $\tilde{i}_b$

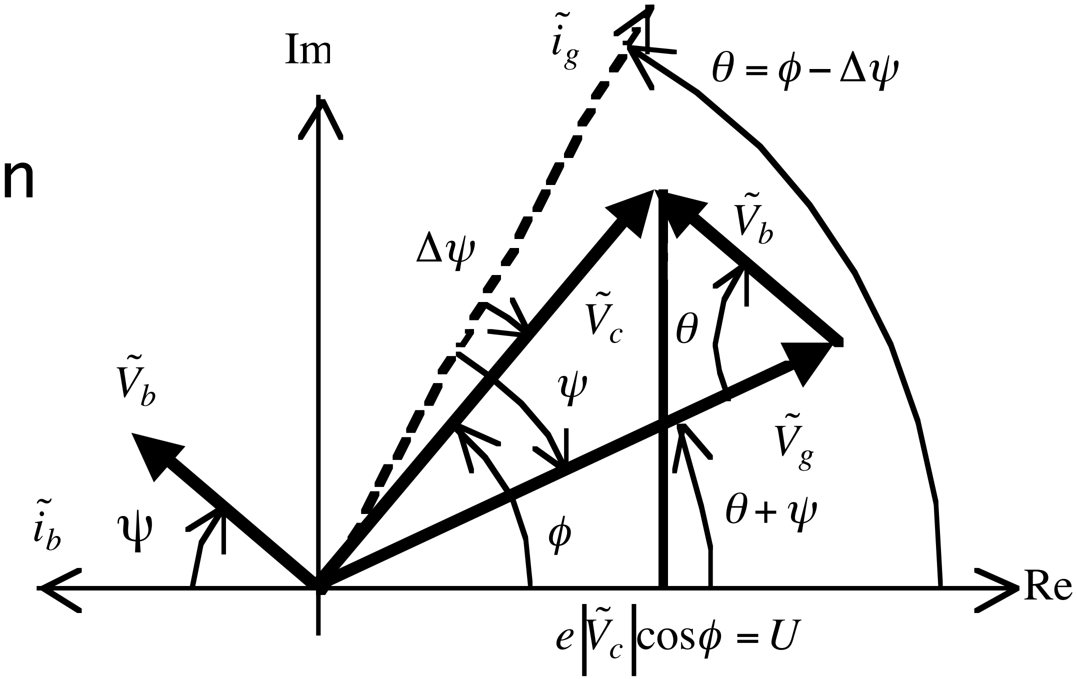
Cavity Voltage

$\tilde{V}_c$

$\psi, \Delta\psi$ Requirement from operation

Required Drive Voltage

$$\tilde{V}_g = \tilde{V}_c - \tilde{V}_b$$



# Application　Longitudinal

Suppression by Acceleration Voltage Modulation

by beam loading of Partial Filling (ESRF)

At commissioning of SPring-8 (1997~8)

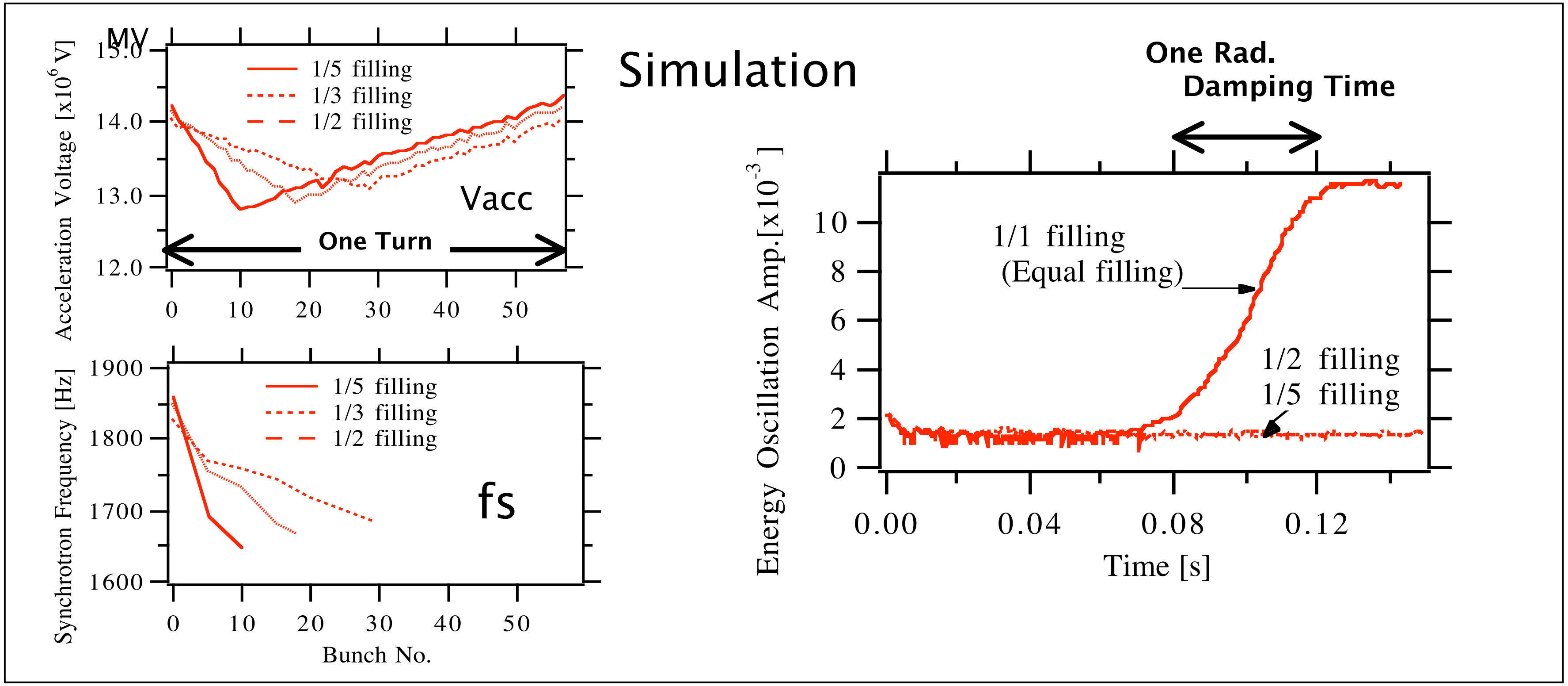
RF Low Level Feedback + beam loading => instability at ~$f_{RF}$

Growth Rate estimation

by Comparison with Simulation

1/5 filling can not suppress => TOO BIG GROWTH-RATE !

=> Other Source but HOM



# Application　Longitudinal

Suppression by Acceleration Voltage Modulation

by Add-on $f_{RF} + f_{rev}$ Accelaration

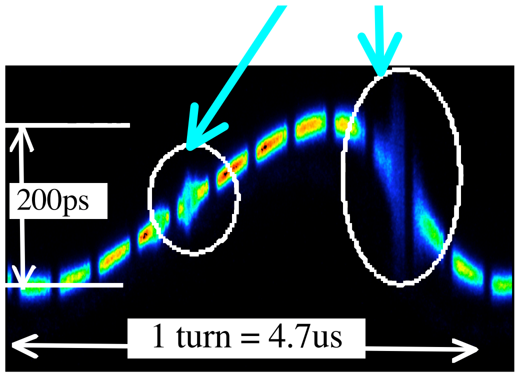
4GeV operation of SPring-8 => Instability

Damping 1/8 of 8GeV (nominal)

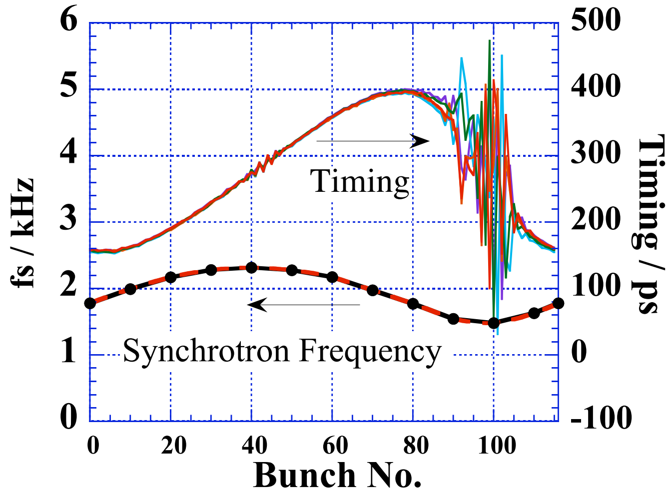
4MV@fRF + 1MV@(fRF+frev)

UVSOR, ETL

unstable




Timing of bunches by Streak Camera




Simulation

Optimization of Filling Pattern

# Application Transverse

## Suppression by Static Chromaticity

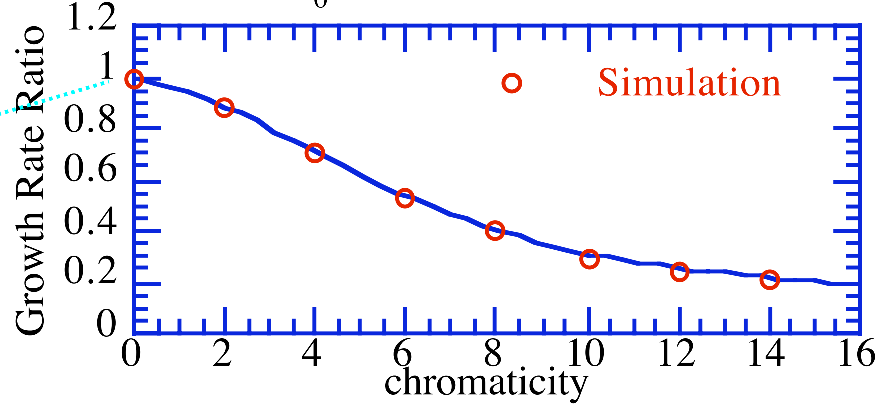
### Reduction Ratio of Growth Rate

$$\omega_f = 2\pi \times 6.3\text{GHz} \quad \omega_s = 2\pi \times 1.7\text{kHz} \quad \omega_0 = 2\pi \times 209\text{kHz}$$

$$\alpha = 1.46 \times 10^{-4} \quad \sigma_\delta = 1 \times 10^{-3}$$

$$\int_0^\infty J_0^2\left(\frac{\omega_0}{\omega_s}\left(\frac{\omega_f}{\omega_0}\alpha + \xi\right)\hat{\delta}\right)\frac{1}{\sigma_\delta{}^2}e^{-\frac{\hat{\delta}^2}{2\sigma_\delta{}^2}}\hat{\delta}d\hat{\delta}$$

$$= \int_0^\infty J_0^2\left(\frac{\omega_0}{\omega_s}(0.7 + \xi)\hat{\delta}\right)\frac{1}{\sigma_\delta{}^2}e^{-\frac{\hat{\delta}^2}{2\sigma_\delta{}^2}}\hat{\delta}d\hat{\delta}$$




## Suppression by AC Chromaticity ( fsyn )

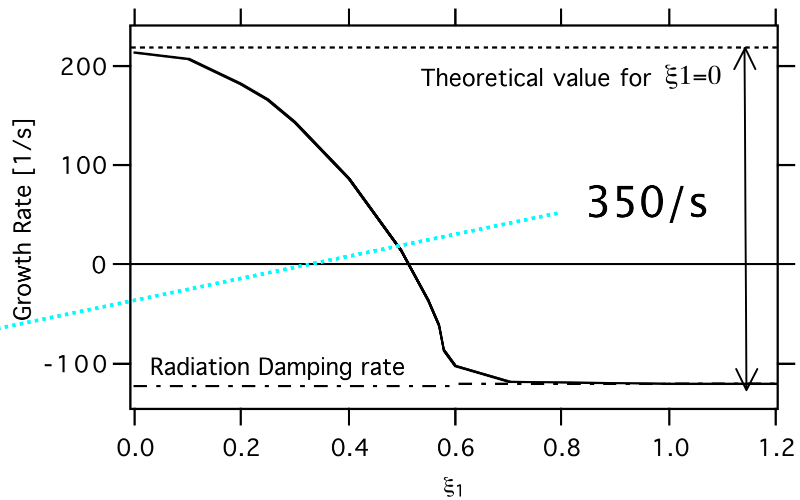
### Introduction of tune spread

$$\xi(t) = \xi_0 + \xi_1 \cos\omega_s t$$

$$\sigma_\nu = \frac{1}{2}\xi_1\sigma_\delta \quad \text{(Gaussian)}$$

$$\xi_1 = 0.6, f_0 = 209\text{kHz}, \sigma_\delta = 1 \times 10^{-4}$$

$$\frac{1}{\tau_L} = \sqrt{\frac{2}{\pi}}2\pi f_0\left(\frac{1}{2}\xi_1\sigma_\delta\right) \cong 2.5 f_0\xi_1\sigma_\delta = 310/s$$



# Single-Bunch

Wake by beam pipe

MAFIA 2-dim, 3-dim

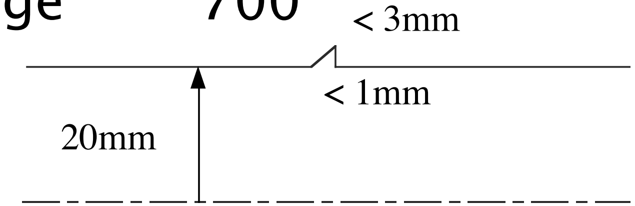
Test Particle Size wake (Green Function)

~ 1mm (meshsize ~ 0.1mm )

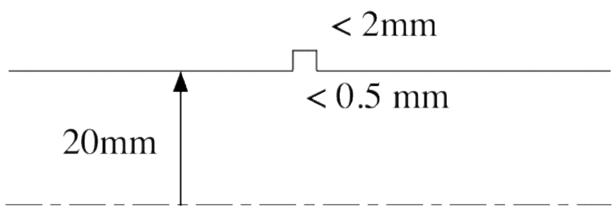
~ 0.2mm (meshsize ~ 0.04mm)

# Vacuum Chamber Shapes

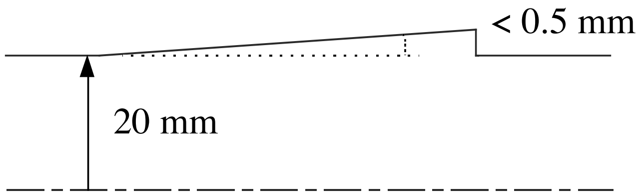
flange 700



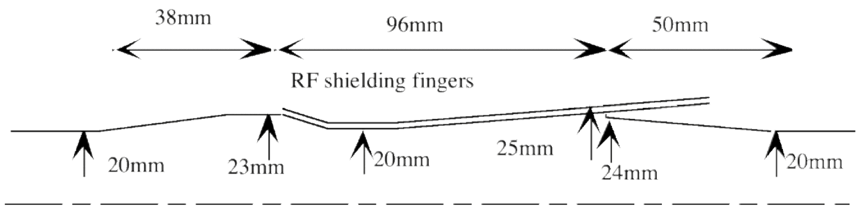

weldment 2000



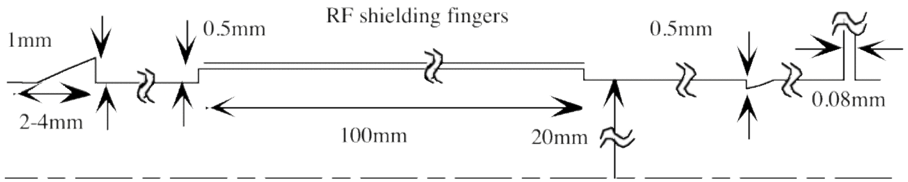

offset 2700




bellows 400




valve 400




Resistive

Aluminum Length 1436m

pipe radius 20mm

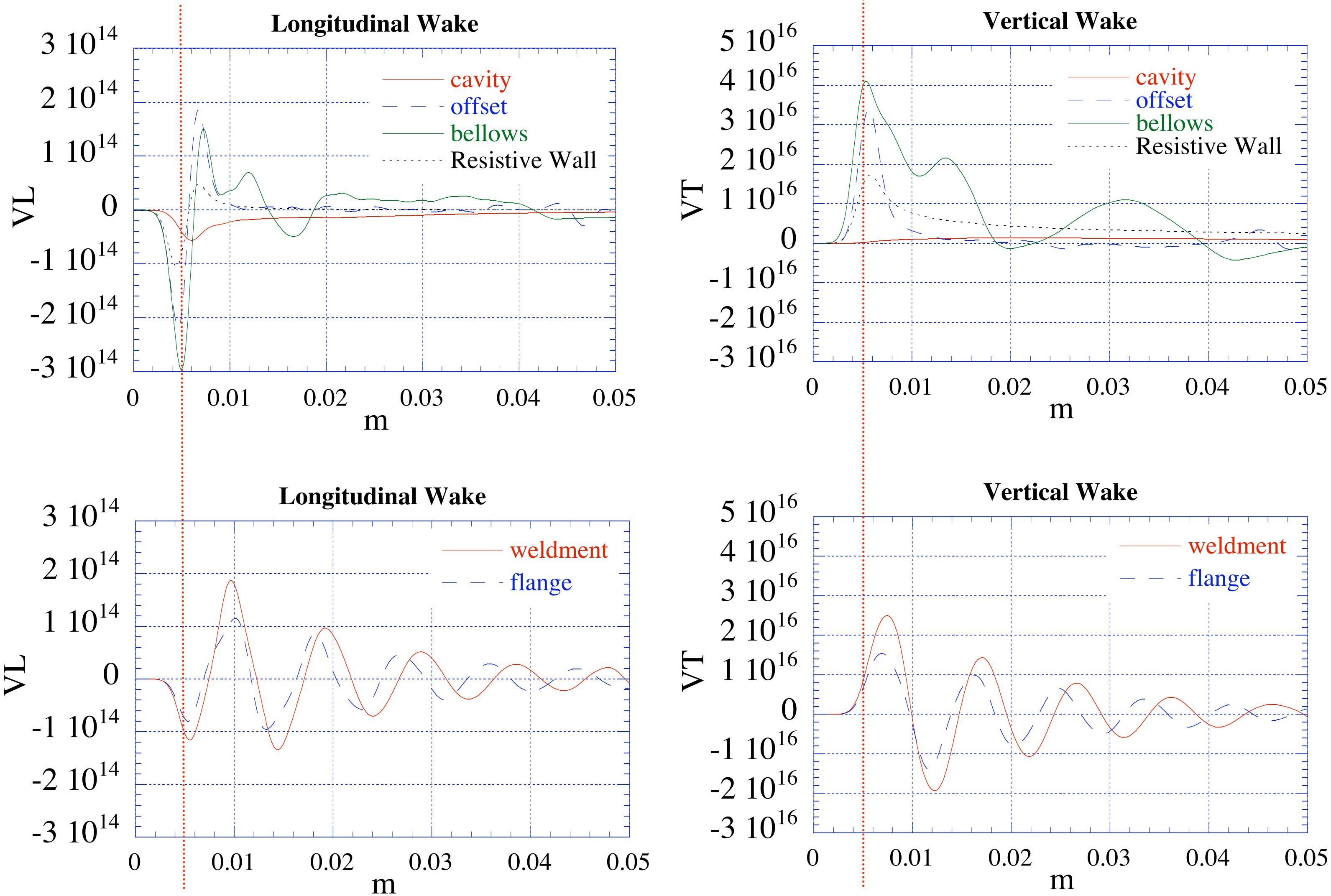
# Wake Function

x Number

x Number x β

bunch center (1mm rms)

bunch center (1mm rms)



# Single-bunch    Equatio of Motion

$$\eta_i = \frac{x_i}{\sqrt{\beta}}$$

$$\theta = \frac{1}{\nu_0} \int^s \frac{ds'}{\sqrt{\beta}}$$

$$\frac{d^2 \eta_i}{d\theta^2} + \left(\nu_0 + \Delta\nu_i\right)^2 \eta_i = \nu_0^2 \beta^{\frac{3}{2}} \frac{F_i}{E_0}$$

$$F_i = e \sum_{j=1}^{N} q_j x_j \frac{d}{ds} W^{\perp}\left(z_j - z_i, s\right) = e \beta^{\frac{1}{2}} \sum_{j=1}^{N} q_j \eta_j \frac{d}{ds} W^{\perp}\left(z_j - z_i, s\right)$$

$$\eta_i = \mathrm{Re}\left[a_i(\theta) e^{i\nu_0\theta}\right] = \frac{1}{2}\left(a_i(\theta) e^{i\nu_0\theta} + a_i^*(\theta) e^{-i\nu_0\theta}\right)$$

$$F_i = \mathrm{Re}\left[f_i(\theta) e^{i\nu_0\theta}\right] = \frac{1}{2}\left(f_i(\theta) e^{i\nu_0\theta} + f_i^*(\theta) e^{-i\nu_0\theta}\right)$$

$$f_i(\theta) = e \beta^{\frac{1}{2}} \sum_{j=1}^{N} q_j a_j \frac{d}{ds} W^{\perp}\left(z_j - z_i, s\right)$$

# Single-bunch Time Average

$$\frac{d^2\eta_i}{d\theta^2} + (\nu_0 + \Delta\nu_i)^2 \eta_i = \nu_0^2 \beta^{\frac{3}{2}} \frac{F_i}{E_0}$$

$$\eta_i = \mathrm{Re}\left[a_i(\theta) e^{i\nu_0\theta}\right]$$

$$F_i = \mathrm{Re}\left[f_i(\theta) e^{i\nu_0\theta}\right]$$

$$\left|\frac{d^2 a_i}{d\theta^2}\right| << \left|2i\nu_0 \frac{da_i}{d\theta}\right|$$

$$f_i(\theta) = e\beta^{\frac{1}{2}} \sum_{j=1}^{N} q_j a_j \frac{d}{ds} W^{\perp}(z_j - z_i, s)$$

$$\frac{da_i}{d\theta} = i\Delta\nu_i a_i + \frac{\nu_0}{2iE_0} \int_{\theta-\pi/\nu_0}^{\theta+\pi/\nu_0} \beta^{\frac{3}{2}} f_i \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)} + \frac{\nu_0}{2iE_0} \int_{\theta-\pi/\nu_0}^{\theta+\pi/\nu_0} \beta^{\frac{3}{2}} f_i^* e^{-2i\nu_0\theta'} \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)}$$

$$\frac{da_i}{d\theta} = i\Delta\nu_i a_i + \frac{\nu_0}{2iE_0} \int_{\theta-\pi/\nu_0}^{\theta+\pi/\nu_0} \beta^{\frac{3}{2}} f_i \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)}$$

# Single-bunch    Time Average

$$\int_{\theta-\pi/\nu_0}^{\theta+\pi/\nu_0} \beta^{\frac{3}{2}} f_i \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)} = \int_{s-\lambda_\beta/2}^{s+\lambda_\beta/2} \beta^{\frac{1}{2}} f_i \frac{ds'}{2\pi} = \frac{1}{2\pi}\frac{\lambda_\beta}{C}\int_{s-C/2}^{s+C/2} \beta^{\frac{1}{2}} f_i ds'$$

$$\int_{s-C/2}^{s+C/2} \beta^{\frac{1}{2}} f_i ds' = \sum_k \beta_k^{\frac{1}{2}} \int_{k-th\ element} f_i ds'$$

$$= e\sum_k \beta_k \sum_j q_j a_j W_k^{\perp}\left(z_j - z_i\right) = e\sum_j q_j a_j \sum_k \beta_k W_k^{\perp}\left(z_j - z_i\right)$$

$$\frac{da_i}{d\theta} = i\Delta\nu_i a_i + \frac{e}{4i\pi E_0}\sum_{j=1}^{N} q_j a_j \sum_k \beta_k W_k^{\perp}\left(z_j - z_i\right)$$

# Single-bunch Difference Equation

$$\frac{da_i}{d\theta} = i\Delta\nu_i a_i + \frac{e}{4i\pi E_0}\sum_{j=1}^{N} q_j a_j \sum_k \beta_k W_k^{\perp}\left(z_j - z_i\right)$$

## Numericaly Stable

$$a_i = r_i e^{i\phi_i}$$

$$r_i^+ = r_i^- + \mathrm{Re}\left[g_i^- e^{-i\phi_i^-}\right]\Delta\theta$$

$$\phi_i^+ = \phi_i^- + \Delta\nu_i\Delta\theta + \frac{1}{r_i^-}\mathrm{Im}\left[g_i^- e^{-i\phi_i^-}\right]\Delta\theta$$

$$g_i^- = \frac{e}{4i\pi E_0}\sum_{j=1}^{N} q_j a_j^- \sum_k \beta_k W_k^{\perp}\left(z_j^- - z_i^-\right)$$

$$\Delta\nu_i = \xi\delta_i^- + \left(amplitude\ \ dependence...\right)$$

Damping by Acceleration

$$a_i^+ = a_i^- - i\frac{eV_a\left(z_i^-\right)}{E_0}\mathrm{Im}\left[a_i^- e^{i\nu_0 u}\right]a_i^- e^{i\nu_0 u}$$

Radiation Excitation

$$a_i^+ = a_i^- + \left(4\frac{\Delta T}{\tau_\beta}\varepsilon_0\right)^{\frac{1}{2}} w_i e^{i2\pi\chi_i}$$

$w_i$ : Gaussian Random Number

$\chi_i$ : Uniform Random Number

# Particle In Cell (PIC) with Wake Field

$$g_i^- = \frac{e}{4i\pi E_0} \sum_{j=1}^{N} q_j a_j^- \sum_k \beta_k W_k^\perp \left( z_j^- - z_i^- \right)$$

$$= \int \sum_{j=1}^{N} q_j a_j \delta\left(z' - z_j\right) \sum_k \beta_k W_k^\perp \left(z_j - z\right) dz'$$

$$= \int \sum_{j=1}^{N} q_j a_j S\left(z' - z_j\right) \sum_k \beta_k W_k^\perp \left(z_j - z\right) dz'$$

$$= \int \sum_{j=1}^{N} \rho_s^a (z') \sum_k \beta_k W_k^\perp \left(z' - z_i^-\right) dz'$$

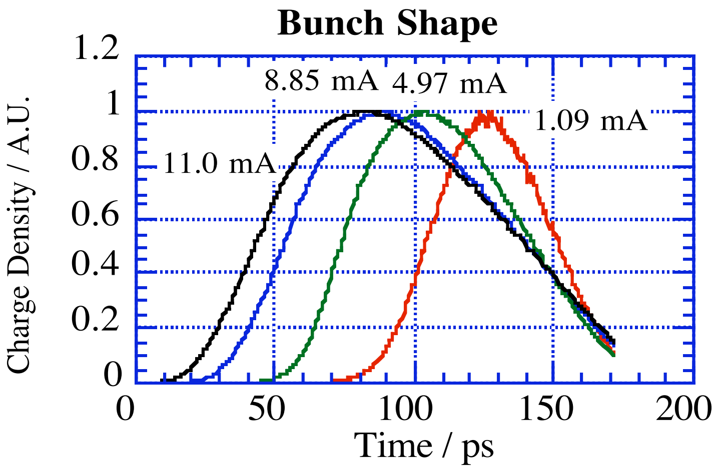
$$S(z) = \begin{cases} -\left(\frac{z}{\Delta z}\right)^2 + \frac{3}{4} & \left(\left|\frac{z}{\Delta z}\right| \le \frac{1}{2}\right) \\ \frac{1}{2}\left(\left|\frac{z}{\Delta z}\right| - \frac{3}{2}\right)^2 & \left(\frac{1}{2} \le \left|\frac{z}{\Delta z}\right| \le \frac{3}{2}\right) \\ 0 & \frac{3}{2} \le \left|\frac{z}{\Delta z}\right| \end{cases}$$

# Comparison with Experiment

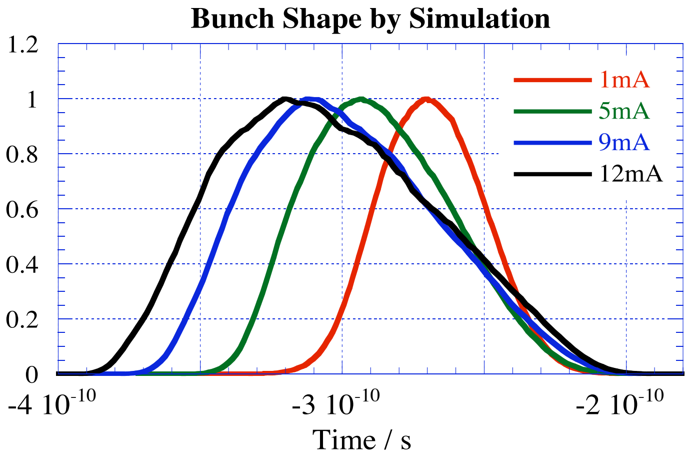
## Longitudinal Bunch Shape

Measured

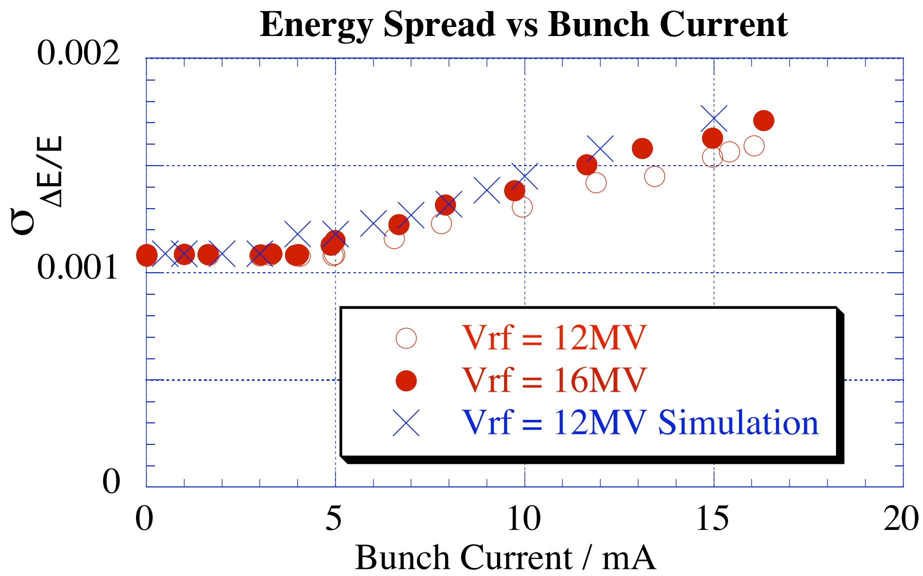
Parasitic loss



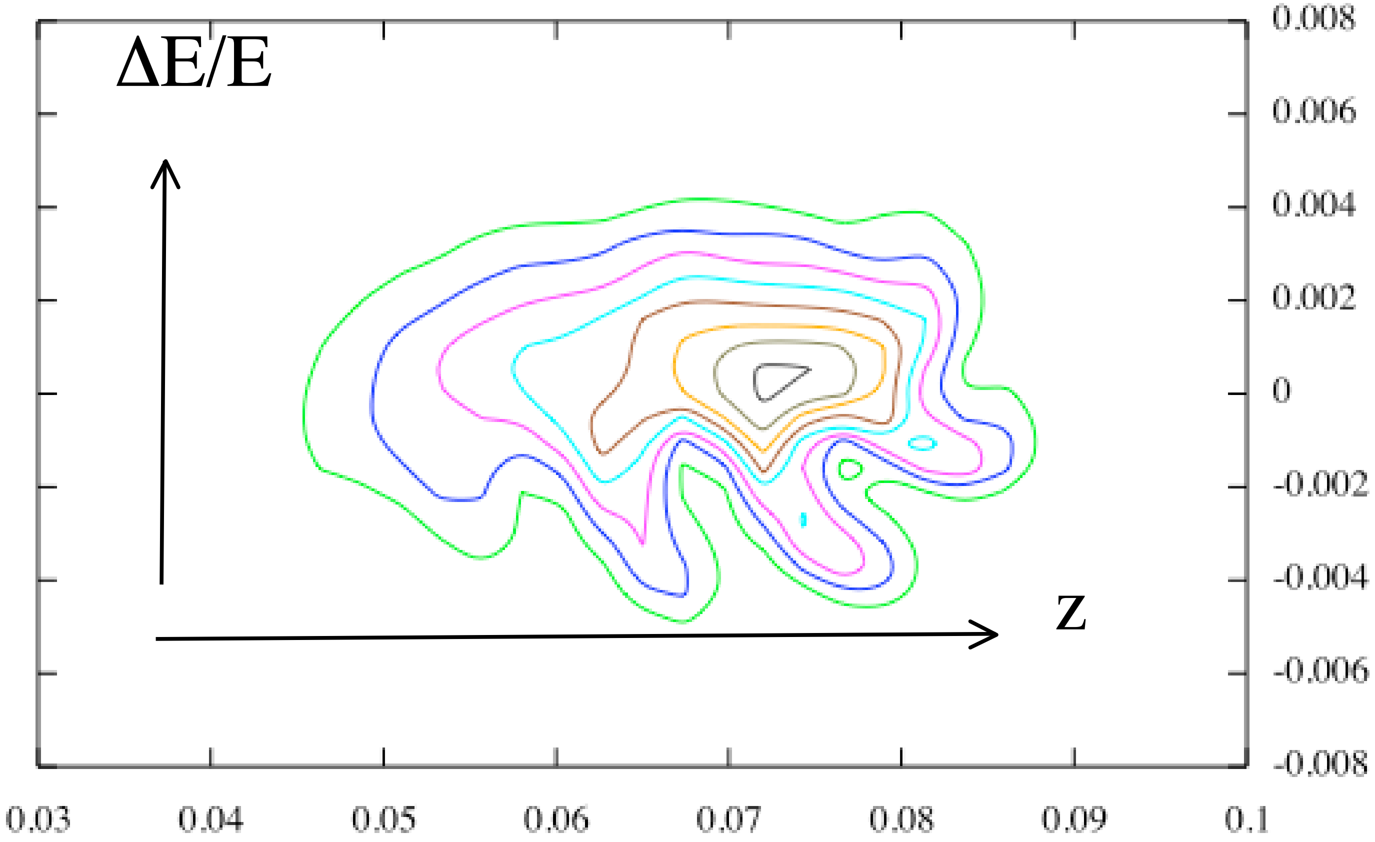

Simulation




## Energy Spread



# Microwave Instability => energy spread driven by high frequency resonance of small grooves(weldment, flange)



# Comparison with Experiment

Vertical Single-Bunch Instabilities

**Prediction by Simulation 1996 (EPAC96 WEP103,WEP104)**

Impedance model( Inductance, Resistance, Cavitylike)

<= Calculated Wake by MAFIA

| Chromaticity | Threshould current (mode-coupling) |
|---|---|
| 0 | 3mA/bunch |
| 4 | 10mA/bunch |

No energy spread increase by model impedance

**Measurement 1998**

| Chromaticity | Threshould current |
|---|---|
| -4.3 | 0.5mA/bunch (m=0 head-tail) |
| 0.24 | 3.5-4mA/bunch (mode-coupling) |
| 4 | > 16mA/bunch |

1.5 times large energy spread at 10mA/bunch

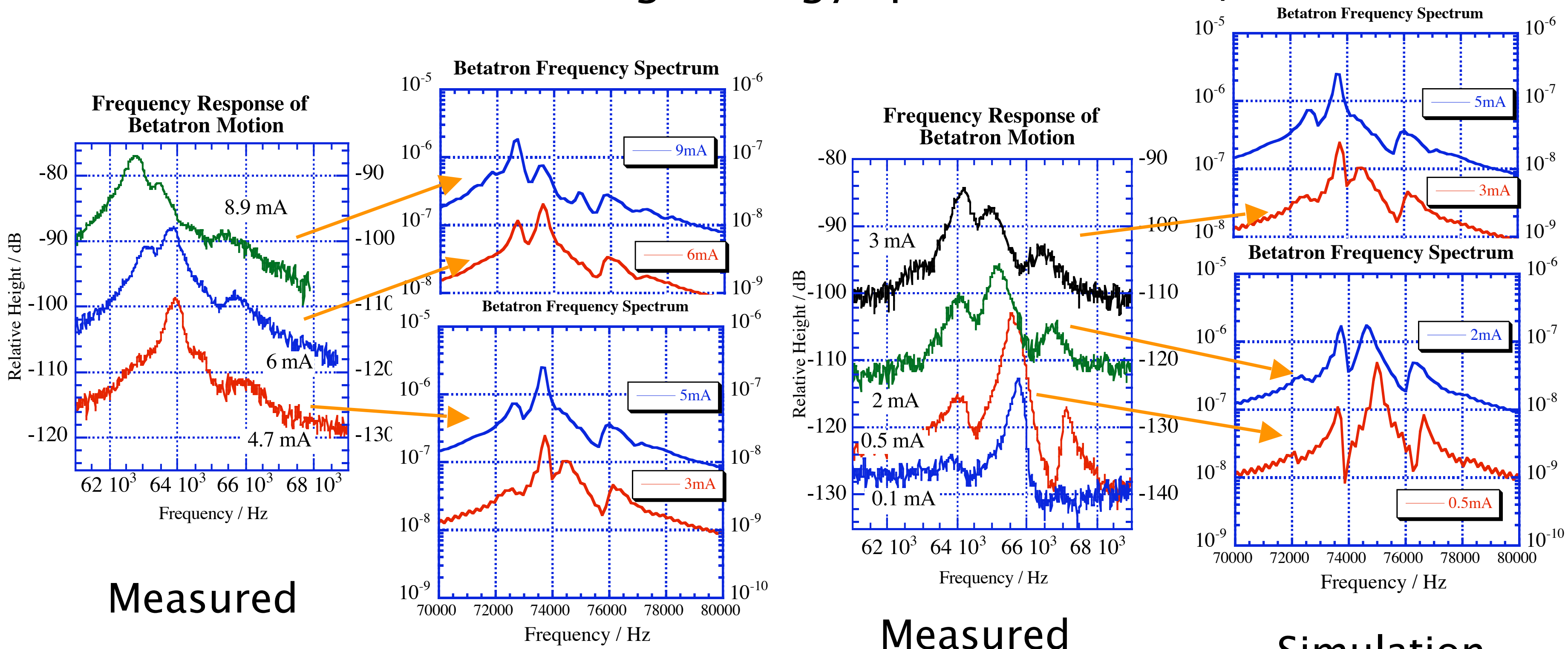
# Comparison with Experiment

Vertical Single-Bunch Instabilities

**Simulation based on Calculated Wake Function**

| | |
|---|---|
| -4.3 | 0.5mA/bunch (m=0 head-tail) |
| 0.24 | 3.5-4mA/bunch (mode-coupling) |
| 4 | > 16mA/bunch |

1.5 times large energy spread at 10mA/bunch



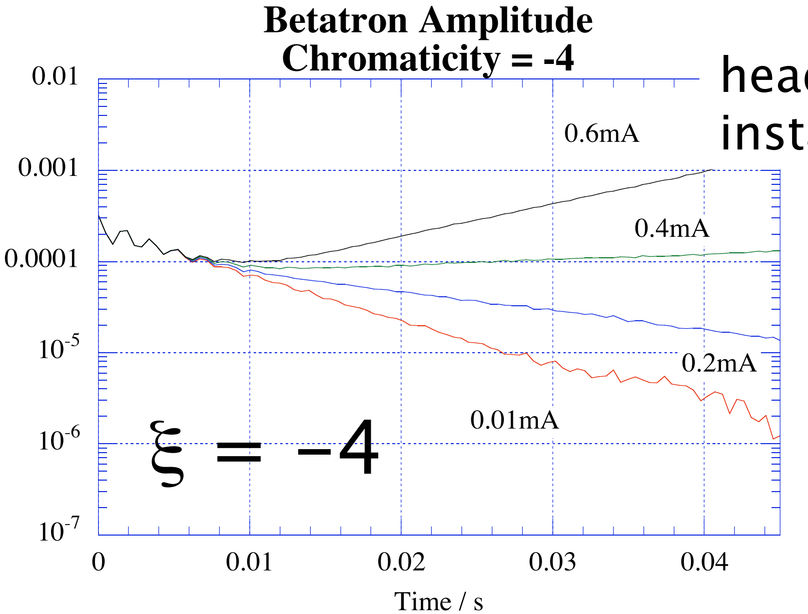

Measured

Simulation

Measured

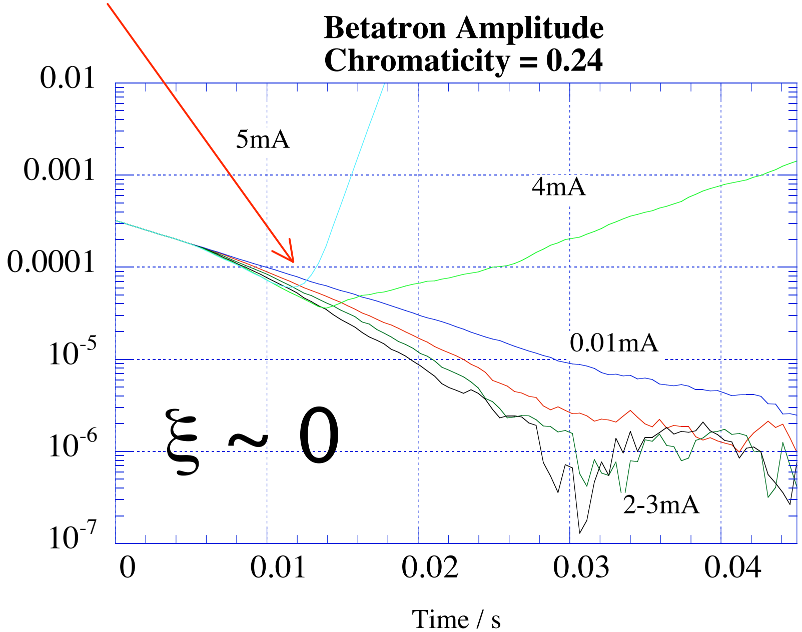
Simulation

# sample of time evolution



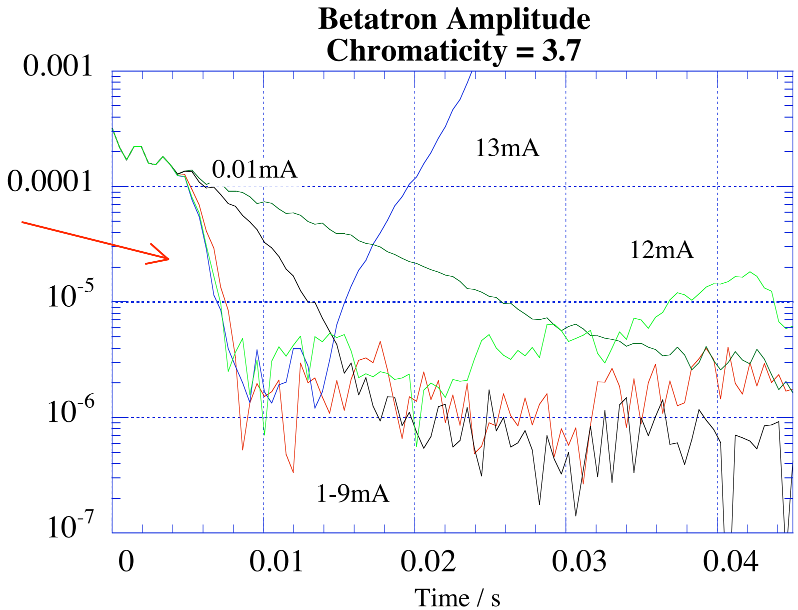

head-tail instability

mode-coupling




head-tail damping

ξ ~ 3.7



## Summary

Multi-bunch simulation (CISR)
Single-bunch simulation(SISR)
developed at SPring-8
Prediction ~ measured

## Next step

Horizontal wake

Electron Cloud, Ion, CSR, ....