Ion Instability in SuperKEKB

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Overview

Introduction

- Positive charged ions in the rest frame oscillate in electric force of electron beam.
- Electron beam and ions start to oscillate the frequency collectively.
- Coupled bunch instability corresponding the frequency is observed.

Ion oscillation in electron beam

Ion with mass M_i near the beam is considered.
Velocity change of ion due to a bunch passage.

$$\Delta v_{i,x(y)} = \frac{e}{M_i} E_{x(y)} \qquad E_{x(y)} = \frac{N_e e}{2\pi\epsilon_0} \frac{x(y)}{\Sigma_{x(y)}(\Sigma_x + \Sigma_y)} \tag{1}$$

where $\Sigma_{x(y)} = \sqrt{\sigma^2_{e,x(y)} + \sigma^2_{i,x(y)}}$ is convoluted beam sizes of beam and ion, and N_e is bunch population.
When bunch train is regarded as an uniform line distribution $\lambda_e = N_e/L_{sp}$,

$$\frac{d^2 x_i}{dt^2} = -\frac{2\lambda_e r_A c^2}{A_i \Sigma_y(\Sigma_x + \Sigma_y)} x_i \qquad \frac{d^2 y_i}{dt^2} = -\frac{2\lambda_e r_A c^2}{A_i \Sigma_x(\Sigma_x + \Sigma_y)} y_i \tag{2}$$

$r_A = r_e m_e / M_A = 1.546 \times 10^{-18}$ m, A_i is atomic number of the ion.

$$\omega^2_{i,x} = \frac{2\lambda_e r_A c^2}{A_i \Sigma_x(\Sigma_x + \Sigma_y)} \qquad \omega^2_{i,y} = \frac{2\lambda_e r_A c^2}{A_i \Sigma_y(\Sigma_x + \Sigma_y)} \tag{3}$$

Ion frequency and unstable mode

Beam parameters

$I_e = 200$ mA, $\varepsilon = 4.6$ nm, $\varepsilon_y = 11$ pm, $\beta_{xy} = 10$ m, $\eta_x = 0.5$ m, $\sigma_\delta = 6.3 \times 10^{-4}$, $\sigma_{x/y} = 381/11$ μm. $N_e = 1.26 \times 10^{10}$, $\lambda_e = 7.1 \times 10^9$ m^{-1}, $L_{sp} = 1.77$ m, $\sigma_{e,x(y)} = \sigma_{i,x(y)}$

$$\omega \propto \sqrt{\sigma_x \sigma_y}$$

Table: Ion frequency and unstable mode.

	H_2	CO	CO_2
A_i	2	28	44
$\omega_{i,y}/2\pi(MHz)$	54.4	14.6	11.6
$\omega_{i,y}/\omega_0$	547	147	117
Mode y	4573	4973	5003
$\omega_{i,x}/2\pi(MHz)$	9.2	2.4	2.0
$\omega_{i,x}/\omega_0$	92	24	20
Mode x	5028	5096	5100

Wake force induced by ion cloud

A bunch with a displacement y_0 passes thorugh an ion cloud. $\Delta t = L_{sp}/c$

$$\Delta v_{i,y} = -\frac{e}{M_i} E_y \Delta t \qquad E_y = \frac{\lambda_e e}{2\pi\epsilon_0} \frac{y_0}{\Sigma_y(\Sigma_x + \Sigma_y)} \tag{4}$$

The ion cloud start to move with Δv_i. The ion interacting with bunches arriving results to oscillate with ω_i,

$$y_i(t) = \frac{\Delta v_{i,y}(y)}{\omega_{i,y}} e^{-\alpha t} \sin \omega_{i,y} t \tag{5}$$

where $\alpha = \omega_i/2Q$ characterize damping of ion oscillation. While bunches arriving after starting the ion motion experience a force from ions

$$\Delta p_y(z) = \Delta y'(z) = \frac{2\lambda_i \Delta s r_e}{\gamma} \frac{y_i(z)}{\Sigma_{x(y)}(\Sigma_x + \Sigma_y)} \tag{6}$$

Action-reaction equality is $N_e m_e \gamma c \Delta y' = \lambda_i L_{sp} M_i \Delta v_i$. $z = -ct$

Wake force induced by ion cloud

$$\Delta p_y(z) = -\frac{2\lambda_i \Delta s r_e}{\gamma \omega_{i,y}} \frac{\Delta v_{i,y}(y_0)}{\Sigma_{x(y)}(\Sigma_x + \Sigma_y)} e^{\alpha z/c} \sin \omega_{i,y} z/c$$
$$= \frac{N_e r_e}{\gamma} \frac{\lambda_i}{\lambda_e} \frac{2 y_0 \Delta s}{\Sigma_{x(y)}(\Sigma_x + \Sigma_y)} \frac{\omega_i}{c} e^{\alpha z/c} \sin \omega_{i,y} z/c \qquad (7)$$
$$= \frac{N_e r_e}{\gamma} w(z) \Delta s y_0$$

Wake force induced by ion cloud, $W = \oint w ds$,

$$W(z) = \frac{c R_S}{Q} e^{\alpha z/c} \sin \omega_{i,y} z/c \qquad z < 0 \qquad (8)$$

The coefficient, R_S/Q

$$\frac{c R_S}{Q} = \frac{\lambda_i}{\lambda_e} \frac{2C}{\Sigma_{x(y)}(\Sigma_x + \Sigma_y)} \frac{\omega_i}{c} \qquad (9)$$

Coupled bunch instability caused by ion cloud

Impedance for ion cloud

$$Z(\omega) = \frac{cR_S}{\omega\left[1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)\right]} \tag{10}$$

Dispersion relation for $exp(-i\mu s/C)$

$$\mu - \mu_x = -i\frac{N_b N_e r_e c}{2\gamma\mu_x}\sum_{p=-\infty}^{\infty} Z[(pN_b + m + \nu_x)\omega_0] \tag{11}$$

Growth rate of the coupled bunch instability ($ReZ(-|\omega|) < 0$: growth)

$$\begin{aligned} T_0/\tau_{G,y} &= -\frac{N_b N_e r_e c}{2\gamma\mu_y}\sum_{p=-\infty}^{\infty} ReZ[(pN_b + m + \nu_y)\omega_0] \\ &= -\frac{N_b N_e r_e c}{2\gamma\mu_y} ReZ[(-N_b + m + \nu_y)\omega_0] \end{aligned} \tag{12}$$

Growth rate of the ion instability in SuperKEKB

$$T_0/\tau_{G,y} = \frac{N_b N_e r_e c}{2\gamma\mu_y}\frac{cR_S}{\omega_i} = \frac{N_b N_e r_e}{2\gamma\mu_y}\frac{\lambda_i}{\lambda_e}\frac{2QC}{\Sigma_{x(y)}(\Sigma_x+\Sigma_y)}$$

$$= \frac{r_e}{\gamma\mu_x}\frac{\lambda_i C^2 Q}{\Sigma_{x(y)}(\Sigma_x+\Sigma_y)} \qquad (13)$$

Unstable mode $H-m$

$$\frac{\omega_i}{\omega_0} \approx (H-m-\nu_y) \qquad (14)$$

These formulae are available for fast ion instability using the ion line density created by bunch train. $\lambda_i = n_i N_{b/tr} = 700\ \mathrm{m}^{-1}$, $n_i = 0.045 N_e P = 5.7\ \mathrm{m}^{-1}$ at $P = 10^{-8}$ Pa.

$$T_0/\tau_{G,y} = 0.01 \qquad (15)$$